Circle True or False or leave blank. (1 point for correct answer, -1 for incorrect answer, 0 if left blank)

1. True   **FALSE** Shifting the graph of a PDF to the left or right changes the standard deviation.

> **Solution:** The standard deviation and variance do not change by shifting the graph (as we saw with the uniform distribution).

2. True   **FALSE** Chebyshev's inequality can help us when $0 < k < 1$.

> **Solution:** When $k < 1$, we have that $1/k^2 > 1$ and hence $1 - 1/k^2 < 0$, so it doesn't help us.

Show your work and justify your answers. Please circle or box your final answer.

3. (10 points)   (a) (4 points) Calculate the standard deviation of $f(x) = \begin{cases} 3x^{-4} & x \le -1 \\ 0 & otherwise \end{cases}$.

The mean is $\frac{-3}{2}$.

> **Solution:** First we need to calculate the mean. The mean is
> $$\int_{-\infty}^{\infty} x f(x)dx = \int_{-\infty}^{-1} x(3x^{-4})dx = \frac{-3x^{-2}}{2}\big|_{-\infty}^{-1} = \frac{-3}{2}.$$
> Then the variance is
> $$\sigma^2 = \int_{-\infty}^{\infty} x^2 f(x)dx - \frac{(-3)^2}{2^2} = \int_{-\infty}^{-1} 3x^{-2} - \frac{9}{4}$$
> $$= \frac{-3}{x}\big|_{-\infty}^{-1} - \frac{9}{4} = 3 - \frac{9}{4} = \frac{3}{4}.$$
> So the standard deviation is $\frac{\sqrt{3}}{2}$.

(b) (4 points) Calculate the standard deviation of $\{0, 0, 3\}$.

Solution: The mean is $\frac{0+0+3}{3} = 1$. The variance is

$$\frac{(0-1)^2 + (0-1)^2 + (3-1)^2}{3} = \frac{1+1+4}{3} = 2.$$

So the standard deviation is $\sqrt{2}$.

(c) (2 points) Let f be a PDF with mean 0 and standard deviation 1. For what value of a can we say that $P(-a \leq X \leq a) \geq \frac{8}{9}$?

Solution: By Chebyshev's inequality, we know that $P(-a \leq X \leq a) = P(\mu - a\sigma \leq X \leq \mu + a\sigma) \geq 1 - \frac{1}{a^2}$. So we need that $\frac{8}{9} = 1 - \frac{1}{a^2}$ so $a^2 = 9$ and $a = 3$.